N64-18297

CODE-1

CR-53354

8p.

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NASA CR-53354 ; TECHNICAL ~~REPORT NO.~~ Rept. 235-3
JANUARY 15, 1964 OTS: $ -- see Cover

prepared for
NASA CONTRACT ~~NO.~~ NASw-682
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON 25, D.C.

014 75 12

ALLIED RESEARCH ASSOCIATES, INC.,
VIRGINIA ROAD CONCORD, MASSACHUSETTS

SYMBOLS

A_{ij}	=	symmetric plasticity coefficient matrix
$\overline{A}$	=	plasticity parameter = $1/2(A_{11}/A_{12})$
B	=	axial rigidity
D	=	flexural rigidity
E	=	elastic modulus
E_s	=	secant modulus
E_t	=	tangent modulus
I	=	distributed moment of inertia per unit width
N	=	membrane stress resultants
$\overline{N}$	=	constant compressive force at buckling
M	=	bending stress resultants
p	=	external pressure
R	=	radius of the sphere
t	=	thickness of the shell
u, v, w	=	displacements
ϵ	=	direct strain variations
χ	=	curvature variations
ν_e	=	elastic Poisson ratio
ν_p	=	fully plastic Poisson ratio
ν	=	instantaneous value of Poisson ratio
σ_{cr}	=	critical buckling stress

ELASTIC AND PLASTIC STABILITY OF GEOMETRICALLY ORTHOTROPIC SPHERICAL SHELLS

Introduction

Stiffening systems offer an attractive means of increasing the structural efficiency of thin spherical shells subject to buckling under external pressure. In this paper, the elastic and plastic stability of geometrically orthotropic stiffened spheres is treated based upon the equations presented in Refs. 1 and 2 for orthotropic cylinders and those in Ref. 3 for thin isotropic spheres.

Derivation of Equilibrium Equations

The stability problem of spheres can be simplified by observing that the buckles are confined to a small portion of the surface and that the buckle wavelength is very small compared to the radius of the sphere. Hence one can study the stability phenomenon in the neighborhood of the buckles using a Cartesian system in the small (Ref. 4).

The strain-displacement relationships are:

$$\epsilon_1 = \partial u/\partial x + w/R \qquad \chi_1 = \partial^2 w/\partial x^2 \tag{1}$$

$$\epsilon_2 = \partial v/\partial y + w/R \qquad \chi_2 = \partial^2 w/\partial y^2 \tag{2}$$

$$\epsilon_3 = 1/2\,[\partial u/\partial y + \partial v/\partial x] \qquad \chi_3 = \partial^2 w/\partial x\,\partial y \tag{3}$$

Equilibrium equations for the buckling problem of a sphere under external pressure, consistent with the strain-displacement relations Eqs. (1) through (3) are:

$$\partial N_x/\partial x + \partial N_{xy}/\partial y = 0 \tag{4}$$

$$\partial N_{xy}/\partial x + \partial N_y/\partial y = 0 \tag{5}$$

$$\partial^2 M_x/\partial x^2 + 2(\partial^2 M_{xy}/\partial x\partial y) + \partial^2 M_y/\partial y^2 + 1/R\,(N_x + N_y) + \bar{N}\,\nabla^2 w + p = 0 \tag{6}$$

In Eqs. (4) to (6), N, M refer to the direct stress and moment resultants produced at buckling, and $\overline{N}$ the constant compressive force = pR/2 at buckling.

The plastic stability theory used herein, following Ref. 3, is based upon a deformation theory of plasticity. Hence the stress-strain relationship for the spherical shells are similar in form in both elastic and plastic ranges. It is advantageous to derive the equilibrium equations in terms of plastic coefficients so that the elastic results can be readily obtained by modifying the coefficients suitably.

The stress resultant-strain relationships in the plastic range for the spherical case are:

$$N_x = B_1 A_{11} (\epsilon_1 + \overline{A}\, \epsilon_2) \qquad M_x = D_1 A_{11} [\chi_1 + \overline{A}\, \chi_2] \tag{7}$$

$$N_y = B_2 A_{11} (\epsilon_2 + \overline{A}\, \epsilon_1) \qquad M_y = D_2 A_{11} [\chi_2 + \overline{A}\, \chi_1] \tag{8}$$

$$N_{xy} = B_3 A_{11} (1-\overline{A})\, \epsilon_3 \qquad M_{xy} = D_3 A_{11} (1-\overline{A})\, \chi_3 \tag{9}$$

Where, B, D are the axial and flexural rigidities in the fully plastic range given by:

$$B_\alpha = E_s t_\alpha/(1-\nu_p^2) \quad ; \quad D_\alpha = E_s I_\alpha/(1-\nu_p^2)\ ; \quad \alpha = 1, 2, 3 \tag{10}$$

with ν_p being the full plastic value of Poisson's ratio (=1/2); I being the distributed inertia per unit width and E_s, the secant modulus.

A_{11} is a component of the plasticity coefficient matrix A_{ij} which for the spherical case has the following form:

$$\begin{pmatrix} \frac{1}{4}(3E_t/E_s + 1) & \frac{1}{2}(3E_t/E_s - 1) & 0 \\ \frac{1}{2}(3E_t/E_s - 1) & \frac{1}{4}(3E_t/E_s + 1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{11}$$

where E_t is the tangent modulus. Finally $\overline{A}$ is in Eqs. (7) to (9) a plasticity parameter given by:

$$\overline{A} = (\tfrac{1}{2})A_{12}/A_{11} = (3E_t/E_s - 1)/(3E_t/E_s + 1) \tag{12}$$

It is clear from Eqs. (7) to (9), with the definitions of various coefficients, that the above relationships can be carried over to the elastic range with the following modifications: $E_t = E_s = E$, the elastic modulus (hence $A_{11} = 1$); B, D now refer to elastic rigidities with ν_e for ν_p; $\overline{A}$ in Eqs. (7) to (9) is to be replaced by ν_e, the elastic Poisson's ratio. With the foregoing modifications all the results that are obtained in the plastic case can be written readily for the elastic case.

By use of the stress-strain and strain-displacement relationships in the equilibrium equations and after using Donnell's technique (Ref. 5), the uncoupled equilibrium equations in terms of u, v and w are obtained as follows:

$$\nabla_B^4 u = -(1+\overline{A})/R\,[B_2(2B/B_1-1)\ \partial^3 w/\partial x \partial y^2 + B_1\ \partial^3 w/\partial x^3] \tag{13}$$

$$\nabla_B^4 v = -(1+\overline{A})/R\,[B_1(2B_2/B_3-1)\ \partial^3 w/\partial x^2 \partial y + B_2\ \partial^3 w/\partial y^3] \tag{14}$$

$$\nabla_B^4[A_{11}\Box_1^4 w + \overline{N}\ \nabla^2 w] + A_{11}(B_1 B_2/R^2)(1-\overline{A}^2)\,\nabla^4 w = 0 \tag{15}$$

where,

$$\nabla_B^4 = B_1\,\partial^4/\partial x^4 + [2\ B_1 B_2/B_3\ (1+\overline{A}) - \overline{A}\ (B_1+B_2)]\ \partial^4/\partial x^2 \partial y^2 + B_2\,\partial^4/\partial y^4 \tag{16}$$

and

$$\Box_1^4 = D_1\ \partial^4/\partial x^4 + \left[2\ D_3\ (1-\overline{A}) + \overline{A}\ (D_1+D_2)\right]\ \partial^4/\partial x^2 \partial y^2 + D_2\ \partial^4/\partial y^4 \tag{17}$$

ORTHOTROPIC SPHERE SOLUTION

An approximate solution for orthotropic stiffened sphere without specific reference to boundary conditions can be obtained under the condition that the wavelength is small relative to a characteristic dimension of the sphere. Choosing

$$w = w_o \sin mx \sin my \tag{18}$$

we obtain from Eq. (15) after minimization

$$\sigma_{cr} = N_{cr}/t_1 = 2\frac{A_{11}}{Rt_1}(1-\bar{A}^2)^{\frac{1}{2}} B_1^{\frac{1}{2}} D_1^{\frac{1}{2}} \left[\frac{(1+D_2/D_1)(1+\bar{A}) + 2D_3/D_1\ (1-\bar{A})}{(1+B_1/B_2)(1-\bar{A}) + 2B_1/B_3\ (1+\bar{A})}\right]^{\frac{1}{2}} \tag{19}$$

The elastic counterpart of Eq. (19) is obtained by replacing the index $\bar{A}$ by ν_e, the elastic Poisson ratio; and with A_{11} reducing to unity, we have:

$$(\sigma_{cr})_{elastic} = \left[3(1-\nu_e^2)\right]^{-\frac{1}{2}} \frac{Et_1}{R}\left[\frac{(1+D_2/D_1)(1+\nu_e) + 2D_3/D_1\ (1-\nu_e)}{(1+B_1/B_2)(1-\nu_e) + 2B_1/B_3\ (1+\nu_e)}\right]^{\frac{1}{2}} \tag{20}$$

Reverting to Eq. (19) we find that $(\sigma_{cr})_{plastic}$ can be written in the following simple form, with the use of Eq. (10) and (11):

$$(\sigma_{cr})_{plastic} = \frac{2}{3}\frac{E_s t_1}{R}\left(\frac{E_t}{E_s}\right)^{\frac{1}{2}}\left[\frac{3(1+D_2/D_1)\ E_t/E_s + 2\ D_3/D_1}{(1+B_1/B_2) + 6\ B_1/B_3\ E_t/E_s}\right]^{\frac{1}{2}} \tag{21}$$

By using an instantaneous value of Poisson ratio ν (Ref. 1) an approximate relation can be obtained for the $(\sigma_{cr})_{plastic}$ as follows:

$$(\sigma_{cr})_{plastic} = [3(1-\nu_e^2)]^{-\frac{1}{2}} \frac{Et_1}{R}\ \eta\left[\frac{3(1+D_2/D_1)\ E_t/E_s + 2D_3/D_1}{(1+B_1/B_2) + 6\ B_1/B_3\ E_t/E_s}\right]^{\frac{1}{2}} \tag{22}$$

where

$$\eta = [(1-\nu_e^2)/(1-\nu^2)]^{\frac{1}{2}}\ (E_s/E)(E_t/E_s)^{\frac{1}{2}} \tag{23}$$

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